HW 2 Solusion

Q-n 11 is 10 pt and 3/3/4

11.

a. .07.

b. .15 + .10 + .05 = .30.

c. Let A = the selected individual owns shares in a stock fund. Then $P(A) = .18 + .25 = .43$. The desired probability, that a selected customer does not shares in a stock fund, equals $P(A') = 1 - P(A) = 1 - .43 = .57$. This could also be calculated by adding the probabilities for all the funds that are not stocks.

Q-n 12 is 10 pt and 2/2/2/2/2

12.

a. No, this is not possible. Since event $A \cap B$ is contained within event B, it must be the case that $P(A \cap B) \leq P(B)$. However, .5 > .4.

b. By the addition rule, $P(A \cup B) = .5 + .4 - .3 = .6$.

c. $P(\text{neither } A \text{ nor } B) = P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - .6 = .4$.

d. The event of interest is $A \cap B'$; from a Venn diagram, we see $P(A \cap B') = P(A) - P(A \cap B) = .5 - .3 = .2$.

e. From a Venn diagram, we see that the probability of interest is $P(\text{exactly one}) = P(\text{at least one}) - P(\text{both}) = P(A \cup B) - P(A \cap B) = .6 - .3 = .3$.

Q-n 29 is 10 pt and 2/2/3/3

29.

a. There are 26 letters, so allowing repeats there are $(26)(26) = (26)^2 = 676$ possible 2-letter domain names. Add in the 10 digits, and there are 36 characters available, so allowing repeats there are $(36)(36) = (36)^2 = 1296$ possible 2-character domain names.

b. By the same logic as part **a**, the answers are $(26)^3 = 17{,}576$ and $(36)^3 = 46{,}656$.

c. Continuing, $(26)^4 = 456{,}976$; $(36)^4 = 1{,}679{,}616$.

d. $P(\text{4-character sequence is already owned}) = 1 - P(\text{4-character sequence still available}) = 1 - 97{,}786/(36)^4 = .942$.

Q-n 30 is 10 pt and 2/2/2/2/2

30.

a. Because order is important, we'll use $P_{3,8} = (8)(7)(6) = 336$.

b. Order doesn't matter here, so we use $\binom{30}{6} = 593{,}775$.

c. The number of ways to choose 2 zinfandels from the 8 available is $\binom{8}{2}$. Similarly, the number of ways to choose the merlots and cabernets are $\binom{10}{2}$ and $\binom{12}{2}$, respectively. Hence, the total number of options (using the Fundamental Counting Principle) equals $\binom{8}{2}\binom{10}{2}\binom{12}{2} = (28)(45)(66) = 83{,}160$.

d. The numerator comes from part **c** and the denominator from part **b**: $\frac{83{,}160}{593{,}775} = .140$.

e. We use the same denominator as in part **d**. The number of ways to choose all zinfandel is $\binom{8}{6}$, with similar answers for all merlot and all cabernet. Since these are disjoint events, $P(\text{all same}) = P(\text{all zin}) + P(\text{all merlot}) + P(\text{all cab}) = \frac{\binom{8}{6}+\binom{10}{6}+\binom{12}{6}}{\binom{30}{6}} = \frac{1162}{593{,}775} = .002$.

Q-n 47 is 10 pt and 2/2/2/2/2

47.

a. Apply the addition rule for three events: $P(A \cup B \cup C) = .6 + .4 + .2 - .3 - .15 - .1 + .08 = .73$.

b. $P(A \cap B \cap C') = P(A \cap B) - P(A \cap B \cap C) = .3 - .08 = .22$.

c. $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{.3}{.6} = .50$ and $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.3}{.4} = .75$. Half of students with Visa cards also have a MasterCard, while three-quarters of students with a MasterCard also have a Visa card.

d. $P(A \cap B \mid C) = \frac{P([A \cap B] \cap C)}{P(C)} = \frac{P(A \cap B \cap C)}{P(C)} = \frac{.08}{.2} = .40$.

e. $P(A \cup B \mid C) = \frac{P([A \cup B] \cap C)}{P(C)} = \frac{P([A \cap C] \cup [B \cap C])}{P(C)}$. Use a distributive law:

$$= \frac{P(A \cap C) + P(B \cap C) - P([A \cap C] \cap [B \cap C])}{P(C)} = \frac{P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)}{P(C)} =$$

$$\frac{.15 + .1 - .08}{.2} = .85.$$